

# A Fast Stochastic Ray-Tracing Method for the 5G Vehicle-to-Vehicle Communication

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**Abstract**—The paper presents the new stochastic ray-tracing method, which enables fast and accurate calculations of distributions of electromagnetic fields for the 5G frequency band, including millimeter waves in a vehicle-to-vehicle propagation channel having random geometrical parameters. The uncertainties of obstacles positions and their sizes, as well as positions of transmitting and receiving antennas, can be considered using the new method. It allows easily to implement uncertainties of antennas phase and magnitude characteristics to the analysis of a stochastic channel model. The method takes advantage of the arbitrary polynomial chaos and the proper transformation of random variables representing geometrical parameters of the propagation channel. The new method is verified using the reference Monte Carlo approach.

**Keywords**—ray-tracing, stochastic simulation, arbitrary polynomial chaos (key words)

## I. INTRODUCTION

The paper deals with calculations of random electromagnetic (EM) fields for vehicle-to-vehicle (V2V) communication. In the paper, a 5G mm-wave propagation is considered. For such frequencies, ray-tracing is used for simulations of an EM wave propagation. It uses the knowledge of the geometry and the material parameters (permittivity, conductivity, roughness) of the propagation channel components. When the vehicle-to-vehicle communication is considered, the consecutive snapshots of the propagation scenario are analyzed as in [1]. The influence of the vehicle velocity on the form of Maxwell equations can be neglected. Consequently, ordinary formulas of the reflection, refraction, and diffraction coefficients can be used for the ray-tracing analysis.

Due to a rapid variation of an amplitude of an electric field of a propagating EM wave in a multipath wireless channel, it is essential to calculate a stochastic distribution of this amplitude. It needs to account for uncertainties in the transmitting and receiving antenna positions. A few centimeters difference in one of these positions for the 5G frequency band could result in over 10dB difference in an amplitude of an electric field.

Stochastic distribution of an electric field amplitude in a V2V propagation channel can be found using the Monte Carlo (MC) approach. Then each realization of random variables of a propagation channel is drawn according to a given joint probability density. Consequently, many repetitions (e.g., more than  $10^5$ ) of a simulation are required to obtain the convergence of the wanted EM field results. The alternative and nowadays more common method of the stochastic EM fields analysis is the general polynomial chaos (gPC) originating from the work of Wiener [2] and finally generalized [3]. It is based on finding the polynomial chaos expansion (PCE) of a random electromagnetic field. In other words, obtaining a PCE meta-model of a random

electromagnetic field is the main goal of the gPC method. The expansion coefficients are used to find the stochastic moments of a random EM field. The gPC is widely used in many engineering applications, including electromagnetism. It can be implemented using an intrusive or nonintrusive approach [3]. The latter does not require to reconstruct the original formulas which are used to calculate deterministic EM fields. It requires much fewer repetitions of a simulation of a propagation scenario comparing with the MC approach [4]. However, it can be ineffective for the microwave frequencies when the number of random variables in the propagation channel is more than several. The application of the gPC method to simulations of propagation of random EM fields was already considered in the literature, e.g. [4, 5]. To the best knowledge of the author, the results presented in this literature are limited to the specific scenarios of a propagation channel. They deal with the analysis of Sobol indices to analyze the random parameters of a propagation channel that have the most influence on an electric field variability. The work in [5] shows that uncertainties associated with the geometry of the propagation channel have the most impact on stochastic distributions of random EM fields. The mentioned papers deal with the nonintrusive approach of the gPC method. As was mentioned earlier, it is hardly ineffective for the 5G frequency band when scenarios with a high number of the geometrical random variables are considered. This inefficiency is a consequence of a rapid variation of EM fields in the geometry domain for the 5G frequency band. From the author's experience, the already known nonintrusive gPC methods [6] are not able to provide accurate results of stochastic electric field distributions for the 5G frequency band in a propagation channel having a high number of the random geometrical variables. Consequently, the much time-consuming MC approach is a better choice.

As the solution to this problem, the author presents the new intrusive approach to the polynomial chaos for stochastic ray-tracing simulations of an EM wave propagation for vehicle-to-vehicle communication.

The new method can be considered as logically and mathematically connected steps that enable to simulate an arbitrary propagation scenario. It ensures accuracy of calculated random EM fields for an arbitrary random geometry of a propagation channel for the 5G frequency band.

In particular, the new method, in an effective way, enables to include uncertainties in antennas positions by an application of proper transformations of the geometrical random variables and the arbitrary polynomial chaos (aPC) [7] to the new random variables. These and other uncertainties of the geometrical and material parameters are covered with the new method. The uncertainties of antennas phase and magnitude characteristics can also be implemented

to the new method by taking advantage of the work presented in [8]. The author assumes in this article that random variables of a propagation channel have uniform probability densities (PDFs) as it is the most common method to model random parameters in a wireless propagation channel.

The paper is organized as follows. Section 2a introduces the block diagram of the new method. The mathematical background of the new method is given in Section 2b. In Section 3 the new method is compared with the MC approach using 2 exemplary vehicle-to-vehicle propagation scenarios. The conclusions are given in Section 4.

## II. THE ARBITRARY METHOD OF STOCHASTIC RAY-TRACING SIMULATIONS FOR A 5G PROPAGATION CHANNEL

### A. The Block Diagram

The new arbitrary method for a ray-tracing simulation of an EM wave propagation for the vehicle-to-vehicle communication is presented by the block diagram shown in Fig. 1.

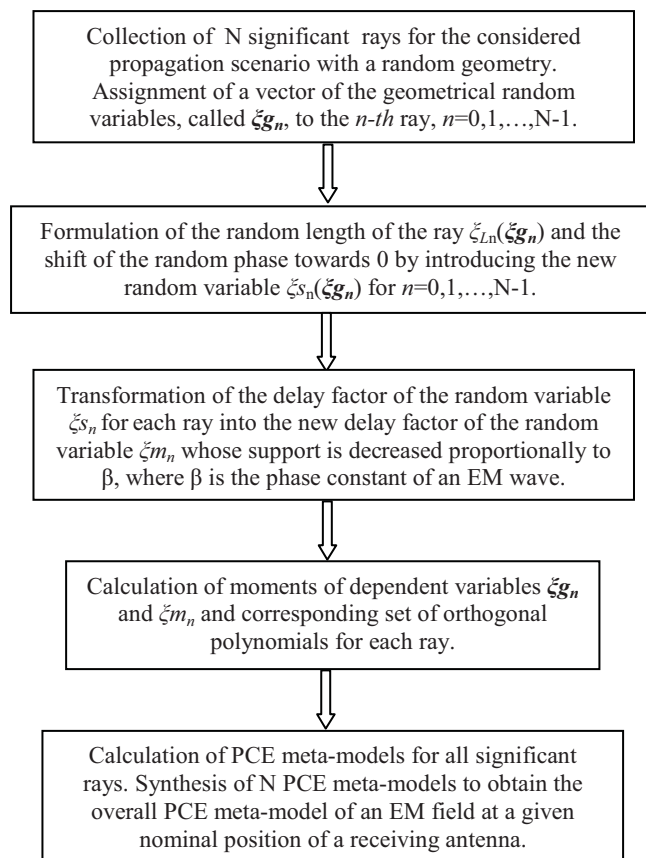


Fig. 1. The block diagram of the new method for the stochastic ray-tracing simulations.

The first step of the method aims to distinguish the most significant rays for the analyzed propagation scenario with random geometry. In the paper, up to three reflections or one diffraction are considered for each ray. The next three steps of the method enable us to minimize the size of the support of the delay factor corresponding to each ray and consequently minimize the number of expansion coefficients of a PCE meta-model of a ray transfer function. Application of these steps enables to significantly shorten the time of required numerical calculations of PCE coefficients due to the substantial decrease of, e.g., integration domain in the Galerkin projection process.

### B. The Mathematical Background

The first step of the method aims only on the collection of significant rays. No additional mathematical transformations are performed in this step.

In the second step, the random length of the  $n$ -th ray is expressed as follows:

$$\xi_{L_n} = f_L(\xi_{\mathbf{g}_n}) \quad (1)$$

where:  $f_L(\xi_{\mathbf{g}_n})$  is a function of a ray length that depends on the propagation scenario geometry. It can be a square root of the sum of squares of the elements in vector  $\xi_{\mathbf{g}_n}$ . The range of the support of  $\xi_{L_n}$  is denoted by  $\langle a_n, b_n \rangle$ . Then, the random phase of each ray is shifted towards 0 using the following form of the delay factor of the  $n$ -th ray transfer function.

$$e^{-j\beta \cdot \xi_{L_n}} = e^{-j\beta \cdot a_n} \cdot e^{-j\beta \cdot \xi_{s_n}} \quad (2)$$

where  $\xi_{s_n}$  is a random variable having PDF with the shape, which is the same as the shape of the PDF of  $\xi_{L_n}$ , however, the range of support of  $\xi_{s_n}$  is  $\langle 0, b_n - a_n \rangle$ .

The random variable  $\xi_{s_n}$  for 5G frequencies has support whose range is too large to integrate the PCE coefficients efficiently. Therefore in the third step of the new method, the periodicity feature of the delay factor (2) is used, and random variable  $\xi_{s_n}$  is transformed into random variable  $\xi_{m_n}$  as follows.

$$\xi_{m_n} = \text{mod}(\xi_{s_n}, \Delta) \quad (3)$$

where:

$$\Delta = \frac{2\pi}{\beta} \quad (4)$$

If the probability density of all the elements in  $\xi_{\mathbf{g}_n}$  is a uniform distribution, then random variable  $\xi_{s_n}$  has a triangular probability function with the maximum value:

$$\max_{p_{-\xi_{s_n}}} = \frac{2}{b_n - a_n}, \quad (5)$$

which occurs for the value of  $\xi_{s_n}$ :

$$\xi_{s_{n\_extr}} = b_n \quad (6)$$

When the probability function of  $\xi_{s_n}$  is denoted by  $p_{sn}(\xi_{s_n})$  then the PDF of random variable  $\xi_{m_n}$  can be calculated as follows.

$$p_{mn}(\zeta m_n) = \begin{cases} \sum_{w=0}^{W-1} p_{sn}(w \cdot \Delta + \zeta m_n) & \text{if } \zeta m_n \leq \xi_{n,th} \\ \sum_{w=0}^W p_{sn}(w \cdot \Delta + \zeta m_n) & \text{if } \zeta m_n > \xi_{n,th} \end{cases} \quad (7)$$

where:

$$W = \text{floor}\left(\frac{b_n - a_n}{\Delta}\right) \quad (8)$$

$$\xi_{n,th} = b_n - a_n - W \cdot \Delta \quad (9)$$

where function  $\text{floor}(x)$  gives the biggest integer number, which is not bigger than  $x$ .

If it is assumed that the material parameters of the propagation channel are deterministic, then a transfer function of each ray depends on random variables  $\xi \mathbf{g}_n$  and random variable  $\zeta m_n$ . The latter appears in the delay factor of the transfer function while  $\xi \mathbf{g}_n$  occurs in reflection, diffraction coefficients as well in spreading (attenuation) factors. When the factor, which dependent on  $\xi \mathbf{g}_n$ , is denoted by  $Hg(\xi \mathbf{g}_n)$ , then the transfer function of a ray is given as follows.

$$HR_n(\xi \mathbf{g}_n, \zeta m_n) = Hg(\xi \mathbf{g}_n) \cdot e^{-j \cdot \beta \cdot a_n} e^{-j \cdot \beta \cdot \zeta m_n} \quad (10)$$

The goal is to find the PCE meta-model of (10). Therefore, the proper orthogonal basis of polynomials for each ray needs to be derived. The polynomials can be denoted as follows.

$$\Psi_{q,p}(\xi \mathbf{g}_n, \zeta m_n) = \sum_{q,p} c_{q,p} \cdot \xi \mathbf{g}_n^q \cdot \zeta m_n^p \quad (11)$$

Now, the PCE expansion of (10) can be written as:

$$HR_n(\xi \mathbf{g}_n, \zeta m_n) = \sum_{q,p} A_{q,p} \cdot \Psi_{q,p}(\xi \mathbf{g}_n, \zeta m_n) \quad (12)$$

where  $A_{q,p}$  is a PCE coefficient, while vector  $\mathbf{q}$  in (11) - (12) is a multidimensional index and is interpreted as in [3]. When  $\xi \mathbf{g}_n = [\xi g_{n,0}, \xi g_{n,1}, \xi g_{n,2}]$  and  $\mathbf{q} = [q_0, q_1, q_2]$ , then  $\xi \mathbf{g}_n^{\mathbf{q}} = [\xi g_{n,0}^{q_0} \xi g_{n,1}^{q_1} \xi g_{n,2}^{q_2}]$ .

Now, the goal is to derive the coefficients  $c_{q,p}$  in (11). The author takes advantage of the algorithm, which is presented in [7]. For this purpose, the author gives the following moment definition.

$$M_{q,p} = I_1 + I_2 \quad (13)$$

where:

$$I_1 = \int_0^{\xi_{n,th}} \sum_{w=0}^{W-1} \xi \mathbf{g}_n^{\mathbf{q}}(g(w, \zeta m_n)) \cdot \zeta m_n^p p_{sn}(g(w, \zeta m_n)) d\zeta m_n \quad (14)$$

$$I_2 = \int_{\xi_{n,th}}^{\Delta} \sum_{w=0}^W \xi \mathbf{g}_n^{\mathbf{q}}(g(w, \zeta m_n)) \cdot \zeta m_n^p p_{sn}(g(w, \zeta m_n)) d\zeta m_n \quad (15)$$

$$g(w, \zeta m_n) = w \cdot \Delta + \zeta m_n \quad (16)$$

It should be noted that  $\xi \mathbf{g}_n(g(w, \zeta m_n))$  is the value of vector  $\xi \mathbf{g}_n$  corresponding to a given value of  $w$  and  $\zeta m_n$ . By analogy to (13) - (15) the expansion coefficients  $A_{q,p}$  are calculated as follows.

$$A_{q,p} = A_1 + A_2 \quad (17)$$

where:

$$A_1 = \int_0^{\xi_{n,th}} \sum_{w=0}^{W-1} \Psi_{q,p}(\xi \mathbf{g}_n(g(w, \zeta m_n)), \zeta m_n) p_{sn}(g(w, \zeta m_n)) d\zeta m_n \quad (18)$$

$$A_2 = \int_{\xi_{n,th}}^{\Delta} \sum_{w=0}^W \Psi_{q,p}(\xi \mathbf{g}_n(g(w, \zeta m_n)), \zeta m_n) p_{sn}(g(w, \zeta m_n)) d\zeta m_n \quad (19)$$

Application of (1) - (19) and the algorithm given in [7] enable us to derive the PCE meta-model of each ray transfer function, which are used then to obtain the overall PCE meta-model of a random EM field as in [8].

### III. SIMULATION EXAMPLES

The formulas given in Section 2 are verified using two simple propagation scenarios for vehicle-to-vehicle communication. The vehicles are placed centrally on opposite road lanes. One of the vehicles is stationary, and the second vehicle is moving. The first simulation example is shown in Fig. 2. The figure shows a 2D scenario with six buildings along the road. The transmitting antenna is placed on the stationary vehicle. The transmitting and receiving antennas are placed 1.55m above the road. The receiving antenna is moving along the red observation line, which is at the center of the road lane. The transmitting antenna is also placed at the center of the road lane. Its nominal position is 1m apart from the nearest building level (see Fig. 2). The complex relative permittivity of the road and walls of buildings are deterministic and equal to  $\epsilon_r = 5 - 0.1j$ . The corresponding roughness of the road and buildings walls is 0.4mm and 1mm, respectively. Reflections from the road and walls of buildings as well as diffraction from buildings edges are considered in the simulation example. The diffraction from the car body is not considered in this paper. The height of the receiving and transmitting antennas positions are deterministic. The other two coordinates are assumed to be random variables distributed uniformly within the square of 0.5x0.5m (green squares in Fig. 2) with the nominal position at the center. The nominal phase characteristic of antennas is  $0^\circ$ , while the magnitude is omnidirectional. To corresponding relative uncertainties of these characteristics are equal to 10%. They are implemented in the simulation, as in [8]. The other uncertainties correspond to the distance of the buildings from the nearest road edge. These uncertainties are 0.2m. The rest of the parameters of the propagation scenario are assumed to be deterministic. The electric field is polarized vertically to the road.

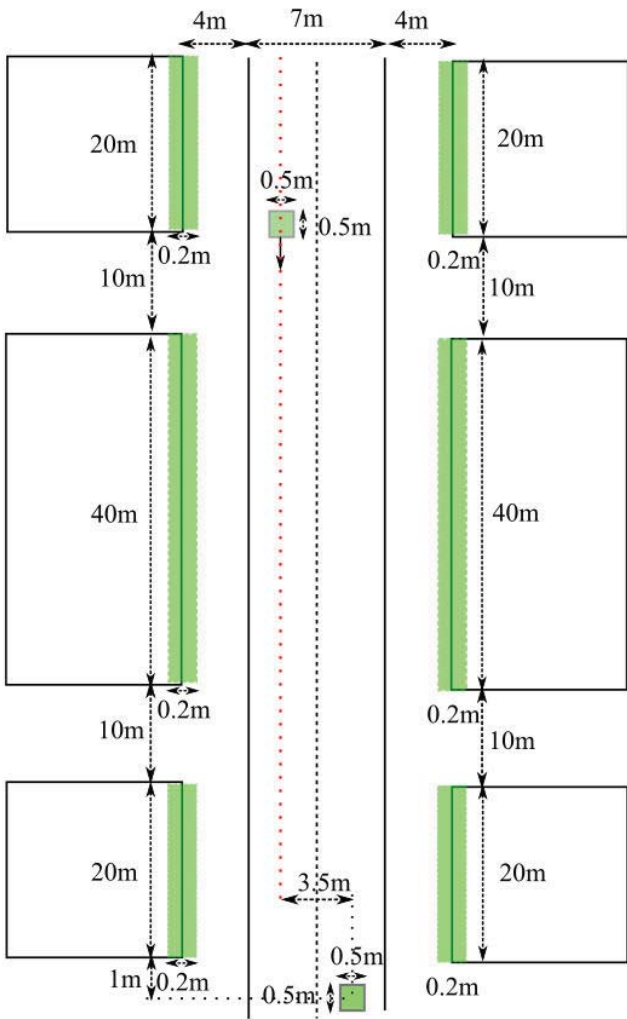


Fig. 2. The propagation scenario of the first simulation example with the red observation line and green uncertainties areas.

Two frequencies are considered in the simulation example, 3.4GHz and 26GHz. In Fig. 3 it is shown the function of the median attenuation of an electric field amplitude calculated along the red observation line using the method described in Section 2 and the MC approach.

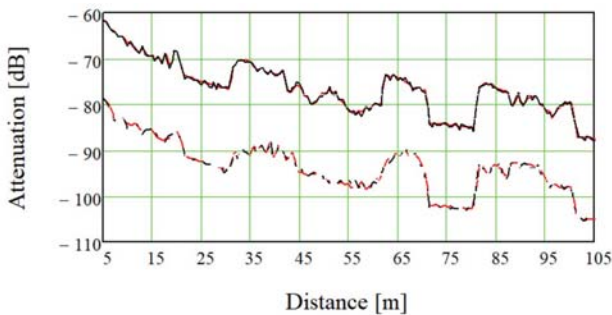


Fig. 3. The median attenuation of an electric field amplitude along the red observation line in Fig. 2 for 3.4GHz and 26GHz obtained using the new method (red curves) and the MC approach (black curves).

An excellent agreement between the new method results and MC results is obtained. The results calculated using (1) – (19) required 1.89 min and 5.08 min of simulation time for 3.4GHz and 26GHz, respectively. The corresponding simulations for the MC approach were completed in 104.14

min and 752.82 min, respectively. A 4-core Intel platform was used for the calculations.

In the second simulation example, a little change in buildings arrangement is made. The rest of the simulation parameters are taken from the first example. The simulation results are shown in Fig. 5 in the same way as in Fig. 3.

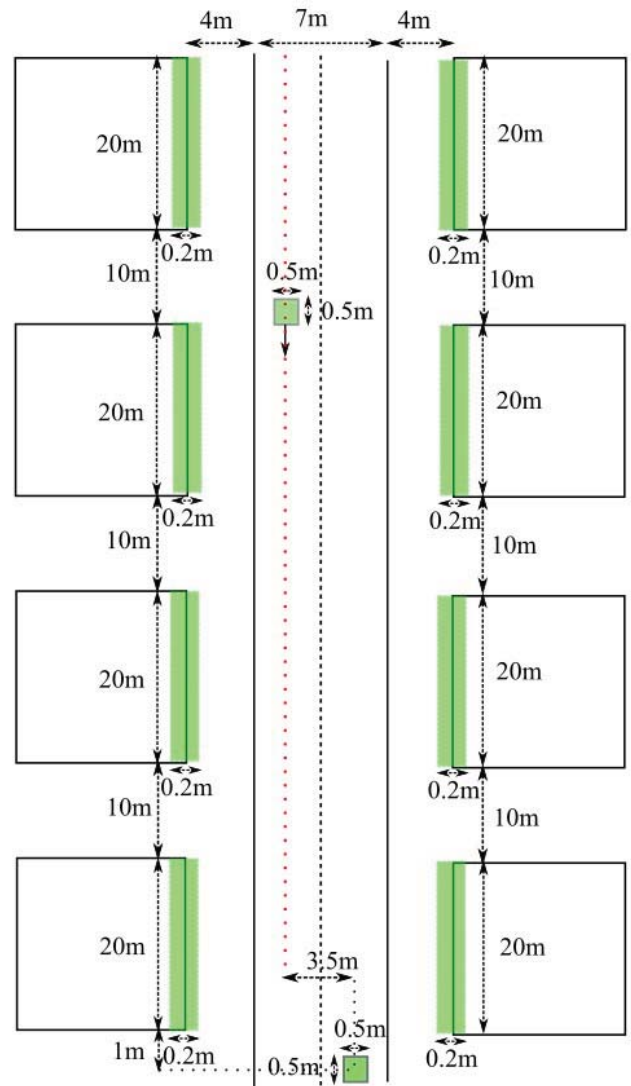


Fig. 4. The propagation scenario of the second simulation example with the red observation line and green uncertainties areas.

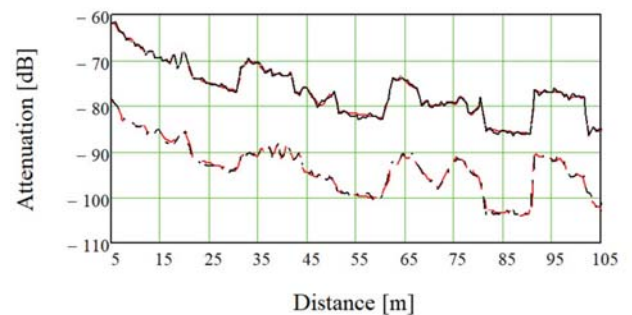


Fig. 5. The median attenuation of an electric field amplitude along the red observation line in Fig. 4 for 3.4GHz and 26GHz obtained using the new method (red curves) and the MC approach (black curves).

Again an excellent agreement between the new method results and MC results is obtained. Simulation times are similar to those observed for the first simulation example.

#### IV. CONCLUSIONS

The main goal of the paper is to introduce the fast and accurate intrusive polynomial chaos method for the calculation of stochastic distributions of EM fields for vehicle-to-vehicle propagation channel in the 5G frequency band. The presented method can account for geometrical uncertainties very efficiently in the mm-wave band what was shown by the two simulation examples. The new method provides a significant speedup of simulation when compared with the MC approach. The author uses the latter as the reference results because the ordinary nonintrusive the gPC methods [3, 6] are not successful when applied to the problem addressed in the paper. It should be stressed that the proposed in the paper transformations of geometrical random variables and application of the arbitrary polynomial chaos enable us to derive the PCE meta-model (12) whose support is significantly reduced. This allows decreasing of the required number of samples of (12) that are necessary to obtain the wanted percentiles of the random amplitude of an electric field. Consequently, the time of MC simulations is much longer than calculations implementing (1) – (19).

#### ACKNOWLEDGMENT

This work was supported by the Ministry of Science and Higher Education.

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